Notional ghosts

Gary Hampson, DownUnder GeoSolutions*

Summary

The notional source method is posed in terms of linear algebra which admits a least squares approach and avoids the original iterative method. We have extended the method to include the new concept of *notional ghosts* which do not need to be related to the notional sources by a linear filter. As a result, the free surface reflection details do not need to be specified. An example is used to demonstrate the new method. Not only does this new approach mean that the reflection process at the free surface may remain unknown, but also that it does not need to be linear. This aspect may turn out to be of considerable use.

Introduction

In a seminal paper, Ziolkowski et al. (1982) introduced the concept of notional sources. Notional sources are conceptual monopole sources that may be combined by linear superposition to determine the pressure at any position. They showed that n notional source signatures can be derived using an iterative method from m near field hydrophone recordings and the geometry of the experiment subject to $m=n$.

Solving for the notional source signatures critically depends upon a favourable geometrical configuration of the hydrophones and airguns (Landro et al., 1991). The original iterative solution (e.g., Parkes et al. (1984)) can compound this issue with divergent behaviour if the system of equations is poorly conditioned.

We formulate the calculation of the notional source signatures in terms of linear algebra in the time domain with full hydrophone and bubble motion. This is solved in a least squares sense which permits $m \geq n$ and allows greater flexibility in geometrical configuration. We observe that it is no longer necessary to assume that the free surface reflectivity is a linear filter. Furthermore, we show how the experiment may be configured to solve for not only the notional sources, but also the notional virtual sources (which we term *notional ghosts*). The details of the free-surface reflectivity are no longer required. The additional effort required to achieve this result is that $m \geq 2n$.

We begin by reformulating the method in terms of linear algebra and then go on to modify the system of equations to accommodate the notional ghosts. Finally we show an example and conclude with a discussion of the potential non-linearity of the source ghost reflection process and its relevance.

The notional source method

Ziokowski et al. (Loc. cit.) reason that for a particular airgun bubble, the hydrostatic pressure modulation in the water from other airguns is equivalent to modulating the pressure inside the airgun bubble while holding the hydrostatic pressure constant. The modified bubble, known as a notional source, may then be treated as a non-interacting monopole source and combined with other sources linearly. This argument permits them to express the i^{th} hydrophone recording as a superposition of n notional sources (p_n) and their ghosts,

$$h_i(t) = \sum_{j=1}^{n} \frac{1}{r_{ji}} p_j\left(t - \tau_{ji}\right) + \alpha \sum_{j=1}^{n} \frac{1}{r_{\mathrm{g}\,ji}} p_j\left(t - \tau_{\mathrm{g}\,ji}\right), \quad (1)$$

in which we closely follow their notation. Here τ and r are time varying due to motion and α is the free surface reflection coefficient. We omit hydrophone sensitivity for brevity. Replacing the time delays and divergences with Dirac delta functions,

$$h_i(t) = \sum_{j=1}^{n} p_j(t) * \left[\delta\left(t - \tau_{ji}\right)/r_{ji} + \alpha\delta\left(t - \tau_{\mathrm{g}\,ji}\right)/r_{\mathrm{g}\,ji}\right]. \quad (2)$$

It can be seen that each hydrophone is a sum of n appropriately ghosted and scaled notional sources. If we denote each time variant convolution as a matrix, $\mathbf{R}_{ij}$, each hydrophone trace, $\mathbf{h}_i$ and each notional signature, $\mathbf{p}_j$, we may write the block matrix form of (2) as,

$$\begin{pmatrix} \mathbf{R}_{11} & \cdots & \mathbf{R}_{1n} \\ \vdots & \cdots & \vdots \\ \mathbf{R}_{m1} & \cdots & \mathbf{R}_{mn} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_n \end{pmatrix} = \begin{pmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_m \end{pmatrix}, \quad (3)$$

or for brevity, $\tilde{\mathbf{R}}\tilde{\mathbf{p}} = \tilde{\mathbf{h}}$. This system is readily solved using conjugate gradient methods for $m \geq n$. In this form it is easily seen that the method rests upon the condition of $\tilde{\mathbf{R}}$ (which only contains geometry and reflectivity). If the condition of $\tilde{\mathbf{R}}$ is marginal, it is much more likely that conjugate gradient methods will succeed than either forming the normal equations or relying on the original iterative approach. In this linear algebraic form it is considerably easier to determine the effects of different geometrical configurations on the condition of $\tilde{\mathbf{R}}$ using, for example, singular value decomposition. Since the ij^{th} time variant ghosting operator is contained in $\mathbf{R}_{ij}$, the free surface reflectivity is easily replaced with any suitable linear operator, such as a rough free surface reflectivity operator.

The notional source method for *notional ghosts*

The source and ghost terms may be completely separated in (2) so that,

$$h_i(t)=\sum_{j=1}^{n}p_j(t)*\delta\left(t-\tau_{ji}\right)/r_{ji}+\sum_{j=1}^{n}a_j(t)*\delta\left(t-\tau_{g\,ji}\right)/r_{g\,ji}, \quad (4)$$

in which p_j is the direct notional source and $a_j=\alpha p_j$ is the result of the notional source reflecting from the free surface. Notice that there is now no mathematical requirement for a_j to have a linear (or any) relationship to p_j. We continue to refer to p_j as a notional source signature, however, we introduce the term *notional ghost* for a_j. It is defined as the effective virtual monopole source that would produce the linear radiated field due to the ghost mechanism acting upon the field from a notional source. Given p_j and a_j, (4) says we can determine the pressure at any point by linear combination.

The block matrix form of (4) is,

$$\left(\begin{array}{ccc|ccc}\mathbf{D}_{11} & \cdots & \mathbf{D}_{1n} & \mathbf{G}_{11} & \cdots & \mathbf{G}_{1n}\\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots\\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots\\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots\\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots\\ \mathbf{D}_{m1} & \cdots & \mathbf{D}_{mn} & \mathbf{G}_{m1} & \cdots & \mathbf{G}_{mn}\end{array}\right)\cdot\begin{pmatrix}\mathbf{p}_1\\ \vdots\\ \mathbf{p}_n\\ \hline \mathbf{a}_1\\ \vdots\\ \mathbf{a}_n\end{pmatrix}=\begin{pmatrix}\mathbf{h}_1\\ \vdots\\ \vdots\\ \vdots\\ \vdots\\ \mathbf{h}_m\end{pmatrix} \quad (5)$$

in which the time variant convolutional blocks for the notional sources and notional ghosts are denoted **D***ij* and **G***ij* respectively. For brevity we write (5) as,

$$\left(\tilde{\mathbf{D}}\mid\tilde{\mathbf{G}}\right)\cdot\begin{pmatrix}\tilde{\mathbf{p}}\\ \hline \tilde{\mathbf{a}}\end{pmatrix}=\tilde{\mathbf{h}} \quad (6)$$

Since there are now twice as many unknowns, it is required that $m\geq 2n$. Similarly to $\tilde{\mathbf{R}}$ in (3), the matrix $\tilde{\mathbf{D}}|\tilde{\mathbf{G}}$ contains geometry, however, it no longer contains reflectivity because it has been absorbed into $\tilde{\mathbf{a}}$. This means that there is no longer a need to specify the free-surface reflectivity. Like $\tilde{\mathbf{R}}$, $\tilde{\mathbf{D}}|\tilde{\mathbf{G}}$ must be sufficiently well conditioned. This is easily explored as a function of geometrical configuration using singular value decomposition.

Conventionally, near field hydrophones are sited about 1 m above the air guns where they detect a linear combination of the up-going notional source signatures and the down-going notional ghosts. The solution of (6) separates the up- and down-going parts of the wavefield. In order to do that, the hydrophones must be located so that the ambiguity of wave direction is resolved by observing the wavefield at different depths. An example of such an arrangement is shown in Figure 1.

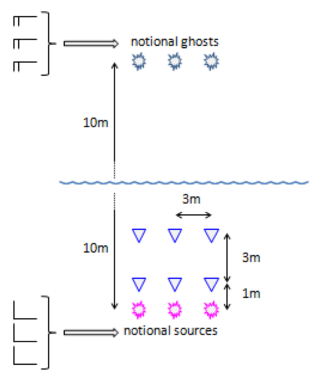

Figure 1 - The configuration of the simple test to demonstrate the derivation of notional ghost functions. The 3-gun array has 6 near field hydrophones. The notional sources are Dirac delta functions, the notional ghosts are a pairs of half amplitude negative Dirac delta functions.

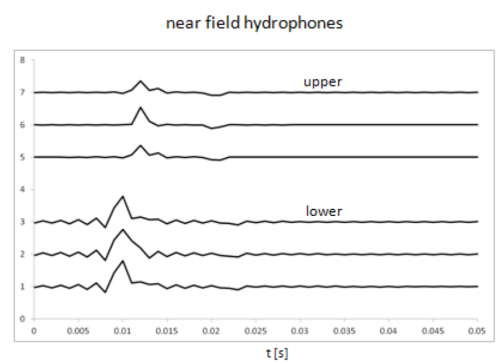

Figure 2 - The modelled near field hydrophone data for the simple 3-gun test in Figure 1.

Results

In order to demonstrate that, given a suitable geometrical configuration and at least 2*n* hydrophones, the method will recover the notional sources and notional ghosts, we use a simple model with a known solution. The configuration, which uses 3 source elements and 6 hydrophones, is shown in Figure 1. The near field hydrophone traces shown in

Figure 2 were simulated using $p_j(t) = \delta(t)$ and $a_j(t) = [\delta(t) + \delta(t - \Delta t)]/2$. The notional ghosts were deliberately designed to overlap the notional source as a test of recovery. The results of solving for the notional sources and notional ghosts are shown in Figure 3. They correspond accurately to the notional sources and notional ghosts used to generate the simulated hydrophone data.

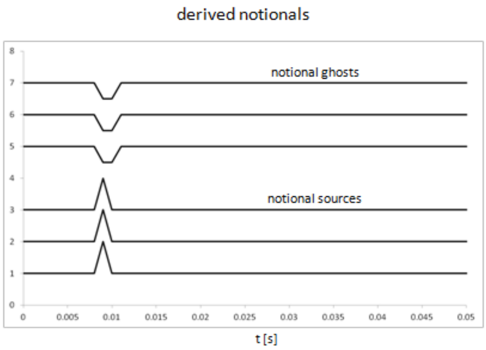

Figure 3 - The resulting notional ghosts and signatures derived from the hydrophone data. They correspond accurately to the notional functions used to generate the model data.

Conclusions

The notional source method has been posed in terms of linear algebra which admits a least squares approach and avoids the original iterative method. It has been extended to include the new concept of *notional ghosts* which do not need to be related to the notional sources by a linear filter. As a result, the free surface reflection details do not need to be specified. An example has been used to demonstrate this new method. Not only does this new approach mean that the reflection process at the free surface may remain unknown, but also that it does not need to be linear. This aspect may turn out to be of considerable use.

Discussion

Traditionally we expect the ghost reflection to be some linearly filtered version of the up-going wavefield (Laws and Kragh, 2002, Orji et al., 2013). At its simplest, this is a scaled delayed copy of the incident field. This linear model is likely to be valid for receiver ghosts where the amplitudes remain in the linear regime. However, in the vicinity of a seismic airgun array, where amplitudes are orders of magnitude higher, there may be good reason to question the validity of the linearity assumption.

Many of the phenomena dealt with in geophysics are, or are assumed to be, linear. This is usually a good assumption and it considerably simplifies analysis. It permits waves travelling in different directions to pass through each other unchanged. It also means that each frequency can be treated entirely independently. In contrast, the behaviour of nonlinear wavefields is much more complex, because it depends upon the amplitudes. Waves no longer pass through each other unchanged and frequencies are no longer entirely independent. Indeed a mark of nonlinearity is that energy is transferred between frequencies, most commonly observed as the generation of harmonics. A second, even more exotic, aspect is that chaotic behaviour such as cavitation and ‘spalling’ occurs.

Are there any indications that these phenomena are associated with air guns?

A relatively simple treatment of non-linear 1D acoustic reflectivity can be found in Wojcik (2004) who develops expressions for a reflection coefficient. These expressions contain a non-linear term which depends upon the amplitude of the wavefield. At low amplitudes they reduce to the traditional linear expression. At higher amplitudes the non-linear term weakens the reflection coefficient and transfers energy between frequencies. A very simple example is shown using the expressions of Wojcik (2004) in Figure 4. It shows a reduction in reflection coefficient magnitude as a function of increase in incident amplitude. This example has assumed a Dirac delta function as the incident waveform. In general the reflection coefficient would be a time series and the reflected wave would interact non-linearly changing the incident wavefield in a zone of interaction proportional in thickness to the duration of the incident waveform. This work suggests that the amplitudes of incident waves are sometimes large enough to cause non-linear reflection.

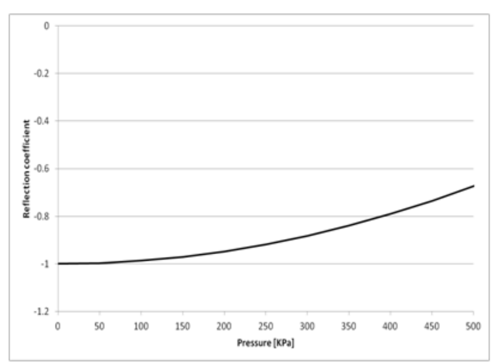

Figure 4 - Non-linear reflection coefficient at the free surface as a function of an incident pressure impulse.

The phenomenon of 'spalling' (e.g., Cushing, 1969) has been noted on numerous occasions in carefully observed air gun experiments and weapons tests. This is the disturbance of the free surface above a strong explosive source in which droplets of water climb up above the surface like long droplet-like fingers. It is the manifestation of cavitation at the free-surface. The water is being pulled back down to its pre-firing position but the tensile strength of the water is insufficient and the water is torn apart. This phenomenon has been referred to as the 'shot effect' by Parkes and Hatton (1986) and is associated with shallower louder airguns. Hatton (2007) discusses this phenomenon and suggests it is associated with a viscous non-linear reflection from the free surface. In that work there is a photograph which is reproduced here as Figure 5.

Landro (2000) described the possibility of ghosted energy conspiring to produce absolute pressures close to zero which results in cavitation. In later work Landro et al. (2011) called this non-linear behaviour *ghost cavitation* and have since reported filming the transient cavitation region forming and disappearing. This phenomenon is described as taking place below the surface affecting the down-going ghosted energy.

Kragh and Combee (2002) conducted an experiment over a salinity reflector in the Orca basin to study ghost behaviour. They showed (their Figure 2, partly reproduced here as our Figure 6) spectra of the salinity reflection for a range of offsets. The receiver ghost notches are approximately 40 dB deep. In contrast, the source ghost (at ~6 m depth) has a notch which is only 10 dB down at the deepest. In this experiment the same sea-state was seen by both the source and the receiver. No satisfactory explanation has been found for the anomalous weakness of the source ghost notch (Ed Kragh, pers. comm.). One might wonder if a frequency dependent Rayleigh reflectivity (Orji et al., 2013) could explain this variation in ghost notch depths. Figure 6 shows two ghost models overlain on their figure. The blue ghost is modeled with an RMS wave height of 0.9 m which seems to roughly match the observed source ghost but fails to match the receiver ghost. The pink ghost uses an RMS wave height of 0.1m which roughly matches the receiver ghost but fails to match the source ghost. Therefore, Rayleigh reflectivity would require that the sea state differed markedly between the source and the nearer offset receivers. This seems unreasonable. An alternative hypothesis might be that the source ghost is behaving non-linearly resulting in a weaker effective ghost.

The complexity of non-linear dynamics is not trivial. We have seen that it includes chaotic behaviour such as the shot effect and cavitation. Its incorporation into the notional source method would add unexplored and probably unwelcomed complexity. In this paper, a method has been presented to extend the notional source technique to include the *notional ghosts*. This provides an alternative pragmatic approach by inverting a sufficiently well conditioned set of observations to estimate not only the notional sources but also the effective wavefield scattered by the free-surface. This does not require the free-surface reflection process to be linear. If non-linear ghost effects are taking place, then the concept of *notional ghosts* may turn out to be of use beyond the conventional linear ghost regime.

Figure 5 - The surface disruption known as the shot effect or spalling above two airguns. (from Hatton (2007) with permission).

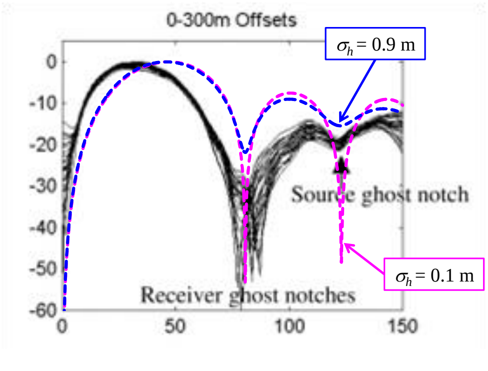

Figure 6 - The amplitude spectra of a number of nearer offsets (after Kragh & Combee (2002) with permission). The source ghost notch is anomalously weak. Two rough sea ghost models are overlain to show that frequency dependent Rayleigh reflectivity fails to explain the anomaly.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Cushing, V. J., 1969, On the theory of bulk cavitation: DTIC Document, Technical Report, http://www.dtic.mil/dtic/tr/fulltext/u2/704616.pdf.

Hatton, L., 2007, An empirical relationship between surface reflection coefficient and source array amplitude, http://www.leshatton.org/Documents/anelastic.pdf.

Kragh, E., and L. Combee, 2000, Using a seismic reflector for resolving streamer depth and sea surface profiles: First Break, **18**, 463–467, http://doi.org/10.1046/j.1365-2397.2000.00463.x.

Landrø, M., 2000, Source signature determination by inversion of ministreamer data: The Leading Edge, **19**, 46–49, http://doi.org/10.1190/1.1438452.

Landrø, M., L. Amundsen, and D. Barker, 2011, High-frequency signals from air-gun arrays: Geophysics, **76**, no. 4, Q19–Q27, http://doi.org/10.1190/1.3590215.

Landrø, M., S. Strandenes, and S. Vaage, 1991, Use of near-field measurements to compute far-field marine source signatures — Evaluation of the method: First Break, **9**, 375–387, http://doi.org/10.3997/1365-2397.1991018.

Laws, R., and E. Kragh, 2002, Rough seas and time-lapse seismic: Geophysical Prospecting, **50**, 195–208, http://doi.org/10.1046/j.1365-2478.2002.00311.x.

Orji, O. C., W. Sollner, and L. J. Gelius, 2013, Sea surface reflection coefficient estimation: 83rd Annual International Meeting, SEG, Expanded Abstracts, 51–55, http://doi.org/10.1190/segam2013-0944.1.

Parkes, G. E., and L. Hatton, 1986, The marine seismic source: Seismology and exploration geophysics, 1st ed.: D. Reidel.

Parkes, G. E., A. Ziolkowski, L. Hatton, and T. Haugland, 1984, The signature of an air gun array: Computation from near-field measurements including interactions — Practical considerations: Geophysics, **49**, 105–111, http://doi.org/10.1190/1.1441640.

Wójcik, J., 2004, Nonlinear reflection and transmission of plane acoustic waves: Archives of Acoustics, **29**, 607–632.

Ziolkowski, A., G. Parkes, L. Hatton, and T. Haugland, 1982, The signature of an air gun array: Computation from near-field measurements including interactions: Geophysics, **47**, 1413–1421, http://doi.org/10.1190/1.1441289.