

## Down/down deconvolution

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### Summary

Up/down deconvolution is a very powerful OBN processing technique which, under the right circumstances, very effectively performs a combination of 3D signature deconvolution and free-surface multiple attenuation on the up-going wavefield. Using an adaptation of the Delft feedback model, we introduce a new idea that we call *down/down deconvolution*. Down/down deconvolution performs a combination of 3D signature deconvolution and free-surface multiple attenuation on the down-going wavefield. As a result, the down-going wavefield can be as effectively deconvolved as the up-going wavefield. This means that the additional shallow reflector coverage that results from down-going mirror-imaging can now be as effectively deconvolved as the more conventional up-going images. We demonstrate our new idea with an example from the North Sea.

### Introduction

Decomposition of wavefield measurements into up-going and down-going components is now routinely performed for data recorded on the ocean bottom. This enables the use of ‘up/down deconvolution’, which performs 3D source signature deconvolution and removes free-surface multiples (Sonneland and Berg, 1987; Amundsen, 2001). Experience shows that provided there is good quality wavefield decomposition, up/down deconvolution is a very effective technique.

Although images made from deconvolved up-going wavefields benefit significantly from up/down deconvolution, they suffer from geometry related disadvantages. In particular, shallower specular reflection points are restricted to a small area around each receiver and the angles of reflection are large. Since ocean bottom receivers tend to be coarsely spaced, the resulting images can be substantially incomplete, particularly in shallower zones. As a result of the wide angles of reflection, significant wavelet stretch occurs which necessitates muting.

By comparison, the down-going wavefield does not suffer from these drawbacks. It uses mirror-imaging which makes use of the virtual receiver instead of the real receiver (Hu and McMechan, 1986). The virtual receiver is located above the free-surface at a height that is equivalent to the depth of the real receiver is below the surface. Consequently the angles of reflection are smaller and specular reflections occur over a wider area around each receiver. As a result, the down-going wavefield is often preferred because it produces better shallow images that have more complete specular reflection coverage. However, unlike the up-going wavefield, there is no powerful deconvolution technique that can be applied to the down-going wavefield. As a result, some combination of conventional methods, which have been adapted to work on ocean-bottom node configurations, is usually invoked.

We begin by describing the up- and down-going scattered wavefield components. We use those to illustrate how up/down deconvolution works before we introduce a new idea in which we use a similarly powerful deconvolution technique on the down-going wavefield. This allows us to combine the advantages of the down-going wavefield with a powerful deconvolution technique. We will refer to this new method as down/down deconvolution and show an example from the North Sea. Finally we will summarise our work and comment on source estimation methods that might be used with our new technique.

### Theory

For simplicity of exposition we consider the scattering problem in the frequency-wavenumber domain. We assume that sufficient independent measurements of the seismic wavefield have been made to provide an unambiguous decomposition into up- and down-going components. Let  $X_0$  denote the impulse response of the Earth in the absence of a free-surface, subscripted by the depth of the source and receiver. We let  $U_0$  and  $D_0$  be the up- and down-going wavefields measured just below the surface. As a result, the convolutional model says that,

$$U_0 = X_0 D_0. \quad (1)$$

Denoting the source wavefield,  $S$ , (including its ghost) we may use the modified Delft feedback model (Berkhout, 1982) shown in Figure 1 to write the up- and down-going wavefields, including the free-surface reflectivity,  $R$ , as the infinite series,

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$$\begin{aligned} D_0 &= S + (RX_0) S + (RX_0)^2 S + \dots \\ U_0 &= X_0 S + X_0(RX_0) S + X_0(RX_0)^2 S + \dots \end{aligned} \quad (2)$$

These may be recognised as Neumann series expansions of,

$$\begin{aligned} D_0 &= \frac{S}{1 - RX_0} \\ U_0 &= X_0 \frac{S}{1 - RX_0} \end{aligned} \quad (3)$$

The common denominator in equations (3) constitutes the reverberation operator (Kennett, 1983). It describes all reverberations between the free-surface and the Earth's heterogeneities.

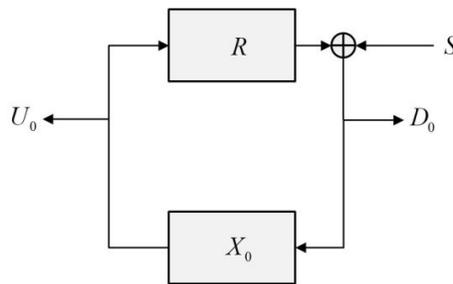


Figure 1 - The Delft feedback model adapted to accommodate up- and down-going wavefields.

Equations (3) are the starting points for a wide range of multiple prediction techniques including surface related multiple elimination (Verschuur, 1991). For simplicity we will present our equations as functions of waves recorded at the surface. However, if we let  $Z = \exp(ik_z z_r)$  be the wavefield extrapolator to downward continue a down going wavefield to depth  $z_r$ , we can always relate surface recordings to those recorded at depth  $z_r$  using the relations,

$$\begin{aligned} D_{z_r} &= Z D_0 \\ U_{z_r} &= Z^* U_0 \end{aligned} \quad (4)$$

The ratio of equations (3),

$$\frac{U_0}{D_0} = X_0, \quad (5)$$

is known as up/down deconvolution. It removes the effects of the free-surface and performs full 3D source signature deconvolution. The result,  $X_0$ , is the impulse response of the underlying Earth as seen from  $z = 0$ . It is customary to transform this to be the response to a monopole source by scaling equation (5) by the factor  $-1/ik_z$  (see Amundsen, 2001). There is an inherent assumption of horizontal layering in this wavenumber formulation (Amundsen, 2001). However, a benefit of this limitation is that we may treat the source and receiver wavenumbers interchangeably. As a result, the best sampled directions (i.e., sources or receivers) may be chosen for the spatial Fourier transform dimensions.

Given  $S$  and  $R$  we could invert either of equations (3) for  $X_0$ . We are most interested in the imaging properties of the down-going wavefield therefore we choose to invert the first of equations (3) so that,

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$$X_0 = \frac{1}{R} \left( 1 - \frac{S}{D_0} \right). \tag{6}$$

With the additional monopole correction this becomes,

$$-\frac{1}{ik_z} X_0 = \frac{1}{ik_z R} \left( \frac{S}{D_0} - 1 \right). \tag{7}$$

Equation (7) is our main result. We refer to it as *down/down deconvolution*. It requires that  $R$  and  $S$  are known.

#### Examples

In order to illustrate down/down deconvolution, in Figure 2 we have constructed a synthetic example for one plane wave. The upper black trace shows the reflectivity used to model pressure and particle velocity in which we used a real airgun signature. The up- and down-going wavefields were then derived from the pressure and particle velocity. The result of up/down deconvolution is shown in grey and the down/down deconvolution is shown as the lower black trace. The recovery of the reflectivity using either deconvolution method demonstrates that our method produces the correct results.

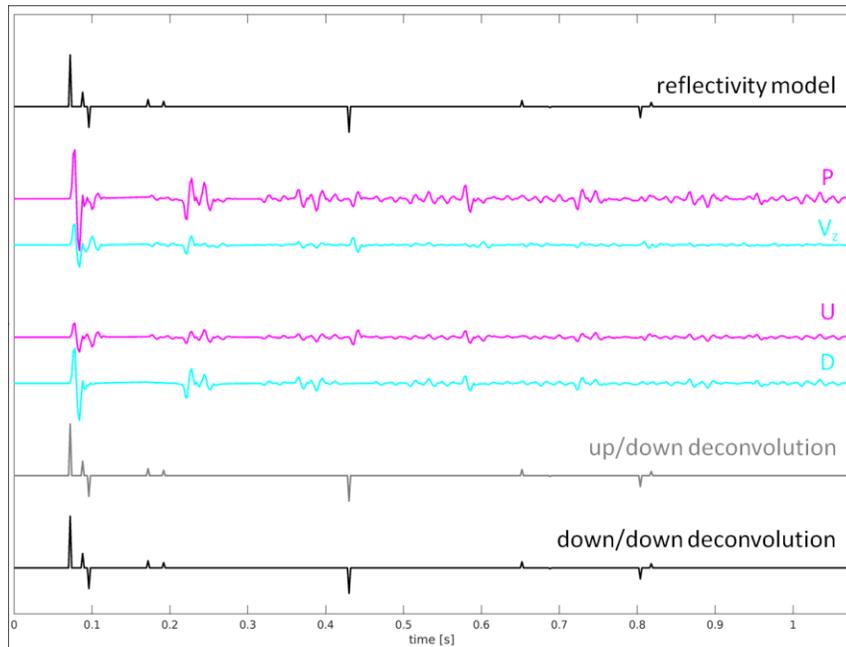


Figure 2 - An illustrative synthetic example for one plane wave.

Figure 3 shows a real example from the North Sea. Figure 3a shows the pre-stack depth migration for up/down deconvolution. The deeper zone is very well deconvolved, however, the shallow zone is poorly imaged. Figure 3b shows the mirror-migration of the down-going wavefield without any deconvolution. Figure 3c is the mirror migration of the down/down deconvolution. The shallow image is considerably improved and the deconvolution has been very effective.

#### Discussion and Conclusions

Images constructed from up-coming wavefields, although very well deconvolved, have geometry related drawbacks. The mirror-imaged down-going wavefield does not suffer from those geometry related disadvantages, however, it lacks a powerful

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deconvolution technique akin to up/down deconvolution. We have shown how up/down deconvolution works using the Delft feedback model and then gone on to introduce a new idea called down/down deconvolution. This new technique inverts the down-going wavefield to remove the effects of the free-surface multiples and perform 3D source wavefield deconvolution producing a result that is theoretically the same as up/down deconvolution.

This new approach needs the free-surface reflectivity and the source wavefield to be known. It will be seen from equation (7) that if  $R$  is assumed to be a scalar or mildly frequency dependent (Orji et al., 2013) then it merely scales the overall result with possibly mild frequency dependence. Therefore the process is relatively insensitive to the free-surface reflectivity.

Without an explicit or implicit source wavefield measurement, the free-surface multiple problem cannot be solved. In up/down deconvolution,  $S$  is implicitly contained in the wavefields that form the numerator and denominator. In SRME the adaptive subtraction step is used to estimate the  $S$  and  $R$  by energy minimisation. For down/down deconvolution, we might consider airgun modelling or the notional source technique (Ziolkowski et al., 1982) or extraction from the down-going wavefield (where the water is sufficiently deep). However, since  $D_0 = S + RU_0$ , then  $S = D_0 - RU_0$ , which is closely related to the ‘cross-ghosting’ method of Soubaras (1986). However, it is worth noting that since  $S$  appears in the numerator of (6) our method is much less sensitive to error than if  $S$  appeared in a denominator.

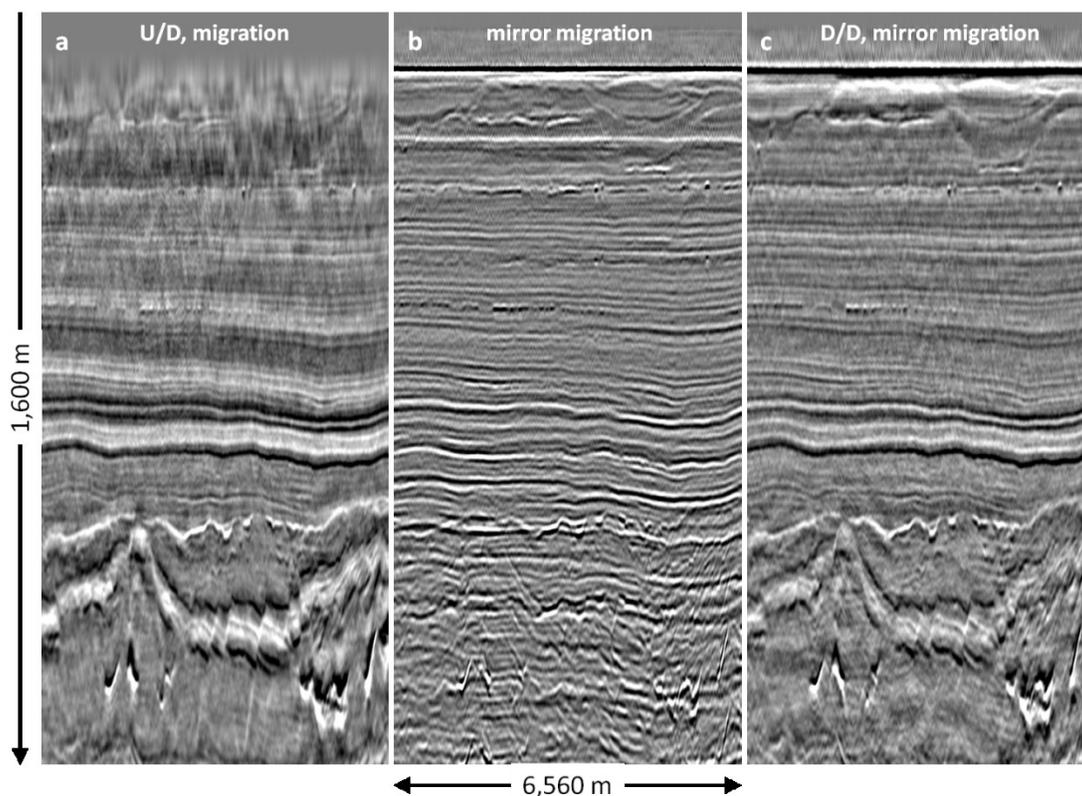


Figure 3 - A stacked PSDM example from the North Sea: a) up-going wavefield with up/down deconvolution, b) down-going wavefield mirror-migrated with no deconvolution and c) down/down deconvolution mirror-migrated. (Data courtesy of AGS and TGS)

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## REFERENCES

- Amundsen, L., 2001, Elimination of free-surface related multiples without need of the source wavelet: *Geophysics*, **66**, 327–341, doi: <https://doi.org/10.1190/1.1444912>.
- Berkhout, A. J., 1982, *Seismic migration: Imaging of acoustic energy by wavefield extrapolation. A. Theoretical aspects*: Elsevier Science Publishers.
- Hu, L., and G. A. McMechan, 1986, Migration of VSP data by ray equation extrapolation in 2-D variable velocity media: *Geophysical Prospecting*, **34**, 704–734, doi: <https://doi.org/10.1111/j.1365-2478.1986.tb00489.x>.
- Kennett, B. L. N., 1979, The suppression of surface multiples on seismic records: *Geophysical Prospecting*, **27**, 584–600, doi: <https://doi.org/10.1111/j.1365-2478.1979.tb00987.x>.
- Orji, O. C., W. C. Sollner, and L. J. Gelius, 2013, Sea surface reflection coefficient estimation: 83rd Annual International Meeting, SEG, Expanded Abstracts, 51–55, doi: <https://doi.org/10.1190/segam2013-0944.1>.
- Sonneland, L., and L. E. Berg, 1987, Comparison of two approaches to water layer multiple attenuation by wave field extrapolation: 57th Annual International Meeting, SEG, Expanded Abstracts, 276–277, doi: <https://doi.org/10.1190/1.1892114>.
- Soubaras, R., 1986, Ocean bottom hydrophone and geophone processing: 56th Annual International Meeting, SEG, Expanded Abstracts, 24–27, doi: <https://doi.org/10.1190/1.1826611>.
- Verschuur, D.J., 1991, Surface-related multiple elimination, an inversion approach: Ph.D. Thesis, Delft University of technology.
- Ziolkowski, A., G. Parkes, L. Hatton, and T. Haugland, 1982, The signature of an air gun array: Computation from near-field measurements including interactions: *Geophysics*, **47**, 1413–1421, doi: <https://doi.org/10.1190/1.1441289>.