The effects of marine currents on seismic data

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Summary

The effects of source and receiver motion have been discussed by a number of authors. However, no attention has been paid to the influence of marine currents, which can attain velocities approaching typical seismic shooting speeds. It is argued that such currents may have a significant effect for OBN analysis, 4D surveys and reconciling data shot in opposing current directions. A theoretical basis is introduced that systematically applies Galilean transformations and wave extrapolations to describe the effects of this motion that includes source and receiver motion. The results are completely consistent with the theory of Doppler (1842) and agree with the theory of Hampson & Jakubowicz (1995). Several methods are described to compensate for the distorting effects of these currents.

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Introduction

Hampson & Jakubowicz (1995) presented an exact theory which described the effects of moving sources and receivers. They showed that the effects are significant, that their theoretical formulation agreed with Doppler (1842) and they described ways to compensate for these effects. As a result, it is now normal to correct for these effects at normal shooting speeds of 2-2.5 m/s. However, the effects of marine *currents* have been neglected so far, even though they often have speeds in the order of 1 m/s or greater. Consequently, we might anticipate that currents will affect seismic data in a somewhat similar manner to source and receiver motion.

I develop a theory for the effects of currents on seismic data using a systematic application of Galilean transforms and wave extrapolation. I show this is completely consistent with the theory of Doppler (1842) and Hampson & Jakubowicz (1995). This approach is used to explore the effects of currents on marine seismic data and to find that under certain circumstances currents significantly distort seismic data. Finally, I propose ways to compensate for these effects.

Theory

Consider a stationary medium overlying a medium moving with velocity u (Figure 1). If an impulse is injected at x_s on the boundary, a semi-circular wavefront will radiate from x_s with velocity, v, into the upper medium. A second semi-circular wavefront will radiate into the lower medium centred on the moving point x_s+ut. With lower medium thickness, z and a stationary receiver on the bottom at x_r, it may be shown that the arrival time is given by

$$t=\frac{\sqrt{v^2\left(x_r-x_s\right)^2+z^2\left(v^2-u^2\right)}-u\left(x_r-x_s\right)}{v^2-u^2}. \tag{1}$$

This configuration models the arrival time at an OBN from a stationary source in a current with velocity u. Let us explore an exaggerated case for illustration alongside a more realistic case. With v=1500 m/s, z=1000 m, u=500 m/s and x_r=0, the results are shown in Figure 2a). The arrival is horizontally sheared with the apex displaced by $-uz/v$. In the realistic case of u=2 m/s (Figure 2b)), the arrival time at 2000 m offset is ~2 ms different to the stationary case. This is significant for adjustments such as OBN location, clock timing and 4D that all rely on direct arrival timing. The effect would be compounded

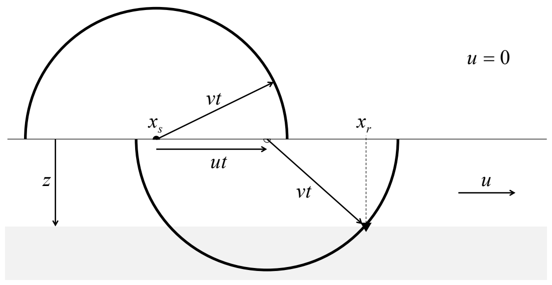

Figure 1 *- An impulse injected between a stationary and a moving medium. The lower wavefront is displaced by ut.*

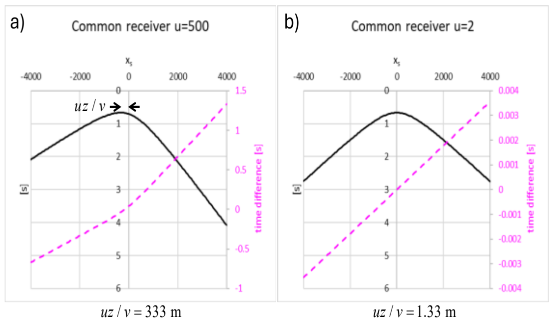

Figure 2 *- The direct arrival time at an OBN taking the current into account. a) The exaggerated case of u=500 m/s. b) A realistic case of u=2 m/s. The dashed line shows the time difference from u=0*

if it had to be reconciled with a second source line acquired in a reversed current direction or comparing 4D surveys acquired under differing current conditions. This simple example shows that there is an effect that deserves investigation.

Galilean Transforms

Galilean transforms are used to map a space-time event between two inertial reference frames, $\mathbf{O}$ and $\mathbf{O}'$, which differ only by the constant *relative* motion between them. Although discussion is limited to 2-dimensional space-time, higher dimensions naturally follow. It is assumed that the coordinates coincide at $t=0$ and that motion is relative. If u is the velocity of $\mathbf{O}'$ w.r.t. $\mathbf{O}$ then the Galilean transformation is given by

$$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -u \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ t \end{bmatrix}. \tag{2}$$

This is a shear transform mapping an event at x in $\mathbf{O}$ to x' in $\mathbf{O}'$, an apparent shift of $-ut$. In terms of a wavefield, this says, that $p(t,x') = p(t,x) * \delta(x+ut)$ which is a time variant spatial shift in the *opposite direction* of u. Since seismic sources, marine currents, the earth and seismic receivers are all inertial reference frames, we may use equation (2) to relate the wavefields in these reference frames.

Source and receiver motion

A source moving at u_s w.r.t. the water will be remapped by the Galilean transform from the moving source reference frame to the stationary water reference frame as $x_s = x_s' + u_s t$, an apparent time variant shift of $+u_s t$. A wavefield in stationary water will be remapped to a receiver moving at u_r w.r.t. the water by the Galilean transform from the stationary water reference frame to the moving receiver frame as $x_r = x_r' - u_r t$, an apparent time variant shift of $-u_r t$. Both results agree with Hampson & Jakubowicz (1995).

The goal now is to describe a systematic model for seismic acquisition that includes currents. The source side and the receiver side will be described separately and for brevity we omit free-surface effects.

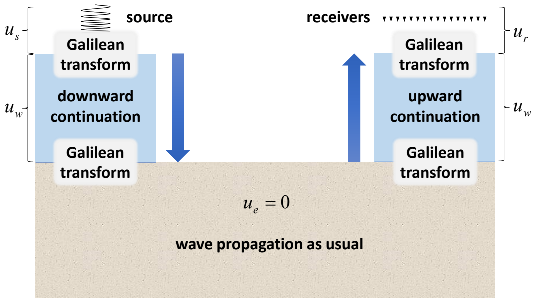

Figure 3 - *A systematic model for moving sources, receivers and currents.*

Source-Water-Earth System

What is the effective source just below the seabed in a stationary earth? I will assume for brevity that there is a single moving body of water. I make use of a combination of Galilean transforms and wave extrapolation operators to describe the source-side of the system shown in the left-hand side of Figure 3. The source signature undergoes transformation into the moving water layer, that wavefield is downward continued to the seabed, where it undergoes another transformation to place the wavefield in the inertial frame of the stationary earth. This wavefield is the effective source. Figure 4 illustrates the source-water-earth system using u_s=0, u_w=500 m/s, z=500 m and v=1500 m, for a vibratory source and the exaggerated motion with a) being the signature at the source, b) being the signature after transformation to the moving water reference frame, c) is the wavefield just above the seabed and d) is the wavefield just below the seabed. In this case the Doppler shift is absent in d) but note the displacement of $+uz/v$ and the asymmetric dips which are the effects of the current. The swept signature is to illustrate any Doppler effects.

Earth-Water-Receiver System

Given an upgoing wavefield just below the seabed, the wavefield recorded at the receivers may be calculated as follows: The wavefield undergoes transformation to the moving inertial frame of the water, which is then upward continued to the depth of the receivers where it undergoes a second transformation into the inertial frame of the receivers. The right-hand side of Figure 3 illustrates the earth-water-receiver system using u_r=0, u_w=500 m/s, z=500 m and v=1500 m/s, with a) being a trace in the earth, b) being the trace after transformation to the moving water reference frame, c) is the wavefield in the water adjacent to the streamer and d) is the wavefield recorded by the streamer. In this case, once more, the Doppler shift is absent in d) but note the displacement of $+uz/v$ and the asymmetric dips which are the effects of the current. The trace used was a random band-limited signal.

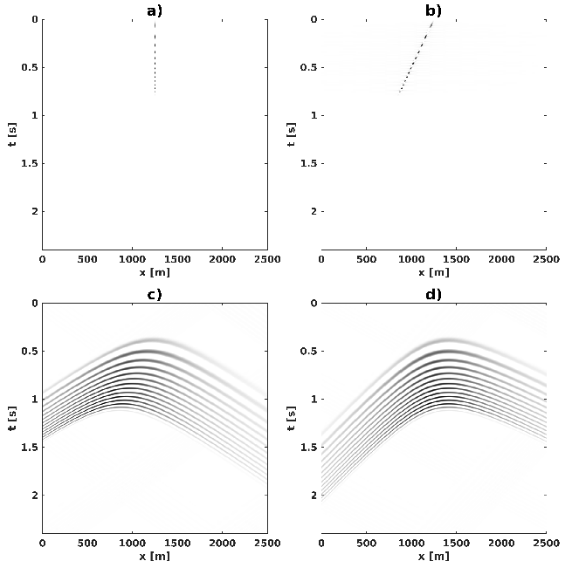

Figure 4 *- The steps in the source-water-earth system: a) a vibratory source signature, b) the signature in the water, c) the source wavefield just above the seabed, and d) the wavefield just below the seabed; the effective source signature.*

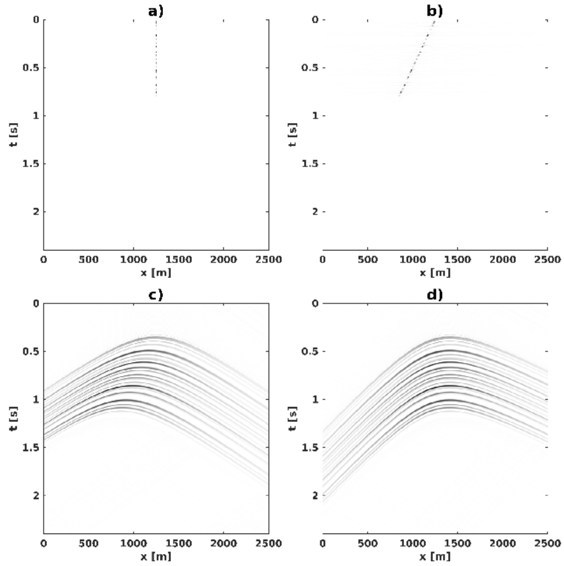

Figure 5 *- The steps in the earth-water-receiver system: a) a trace in the earth, b) the trace in the water, c) the wavefield adjacent to the streamer, and d) the wavefield recorded by the streamer.*

Clearly a stationary streamer or vibrator is unusual, however, by only having the water moving, the effects of the current can be shown in isolation. In both cases the wavefield is asymmetrical and displaced. The displacement is proportional to water depth and current speed, and so of more significance in deep water with stronger currents. The effects will be compounded if two adjacent profiles were acquired in opposing current directions.

This systematic approach works with time dependent waveforms and depth extrapolation. An alternative approach uses a moving 2-way wave equation to extrapolate space dependent waveforms over time which has the advantage of accommodating multiples and facilitating FWI.

The Doppler effect
The Fourier domain equivalent to the Galilean transform is

$$\begin{bmatrix} \omega' \\ k_x' \end{bmatrix} = \begin{bmatrix} 1 & -u \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \omega \\ k_x \end{bmatrix}. \tag{3}$$

This is a shear transform which maps an event at ω in $\mathbf{O}$ to ω' in $\mathbf{O}'$; a frequency shift of $-uk_x$. In terms of a wavefield, this says, that $P(\omega',k_x') = P(\omega,k_x) * \delta(\omega + uk_x)$ which is a wavenumber variant temporal frequency shift in the *opposite direction* of u. This is the Doppler (1842) effect noted by Hampson & Jakubowicz (1995). As depth extrapolation of wavefields is easily performed by multiplication with $\exp(ik_z z)$ we can write the systematic model in the Fourier domain as

$$L_{r|w} e^{ik_z z} L_{w|e} R L_{e|w} e^{ik_z z} L_{w|s} S\ , \tag{4}$$

in which $L_{b|a}$ is the Fourier Galilean transform (3) from a to b, R is the solid earth's impulse response and S is the source signature. It being understood that (4) should be evaluated right-to-left. Temporal and spatial frequencies are not changed by $\exp(ik_z z)$. Furthermore, nor does the Galilean transform change spatial frequencies. In contrast, the earth's response typically changes spatial frequencies due to scattering and heterogeneity. As a result, the effects of the system need to be split into the source-side and the receiver-side. Using the notation of (4), the source and receiver sides are respectively

$$L_{r|w}L_{w|e}, \quad L_{e|w}L_{w|s}. \tag{5}$$

Evaluating (5) for the frequency entering the earth and the frequency recorded at the receiver respectively, we find that

$$\omega_e = \omega_s + u_s k_s, \quad \omega_r = \omega_e - u_r k_e, \tag{6}$$

which has assumed that $u_e = 0$. The subscripts indicate in which reference frame the variables apply. Equation (6) shows that there is <u>no Doppler effect due to the current.</u> Since on the receiver side, $k_e = k_w = k_r$ we may replace the second of equations (6) with $\omega_r = \omega_e - u_r k_r$. Noting that $k_s = \omega_s p_s$ and $k_r = \omega_r p_r$, in which $p = \partial t / \partial x$ w.r.t. to the relevant inertial frame, we may write

$$\omega_e = \omega_s\left(1 + u_s p_s\right), \quad \omega_r = \omega_e \big/ \left(1 + u_r p_r\right). \tag{7}$$

Substitution of the first of these into the second shows that

$$\omega_r = \omega_s \frac{1 + u_s p_s}{1 + u_r p_r}, \tag{8}$$

which is the Doppler effect, consistent with Doppler (1842) and Hampson & Jakubowicz (1995).

Compensating for the effects of motion

How might we compensate for the effects due to the motion of the sources, receivers and the currents? One approach is to perform common shot migration taking the motion into account. In this approach, migration would include the source signature in the forward source propagation along with a moving 2-way wave equation. This has the advantage that it could also be used in least squares migration or full waveform inversion where it can properly accommodate multiples.

However, since the Doppler effect is due to source and receiver motion, those distortions may be corrected using Hampson & Jakubowicz (1995) as a first step. In doing so there remain distortions due to the currents, that is, the displacement and asymmetry in the wavefields noted in Figure 4d) and Figure 5d). The source side is a deconvolution problem using the effective source wavefield (Figure 4d)), while the receiver side may be resolved by applying the adjoint of the receiver side model, $\left(L_{r|w} e^{ik_z z} L_{w|e}\right)^{\mathrm{H}}$. The adjoint is chosen because the inverse of the wave extrapolator is a growing exponential function.

Conclusions

A theoretical model for the inclusion of marine currents into the motion effects on seismic data has been presented. It has been shown that currents distort the seismic wavefield, however, it has been found that they do not contribute to the Doppler shift, rather, they displace and shear the wavefield. On the source side this means that at the seabed a modified *effective* source wavefield, which is displaced and asymmetric, propagates into the solid earth. On the receiver side the received wavefield is also similarly displaced and asymmetric. The displacement for each side is $u_w z / v$ where z is the water depth. The apparent velocity asymmetry asymptotes to $u_w \mp v$. Two approaches have been proposed to compensate for these effects both using modified wave equations, one incorporates the corrections proposed by Hampson & Jakubowicz (1995). Finally, it has been demonstrated that this new theory is completely consistent with Doppler (1842) and Hampson & Jakubowicz (1995).

References

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